Chapter 1

A Spatiotemporal Coupled Lorenz Model drives Emergent Cognitive Process

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Proposed emergent system which is n subsystems (n spatiotemporal coupled Lorenz models plus n abstract coincidence detector models) are connected to the external input of the n neurons mutually connected neural networks through the connecting weight with external stimuli. Proposed system is an auto-correlation type associative memory model. Even though having no learning synapse weight systems in the model that shows several autonomous dynamics of retrieving embedded vectors, which looks like "chaotic itinerancy" by exciting external stimuli from the subsystems.

1.1 Introduction

The mutually connected neural networks are often used as a method of artificially making the associative memory that is one of the higher-order functions of human brain's behaviors. A system known well is the Hopfield network model proposed by J. J. Hopfield in 1982 [1]. In that case of using the Hopfield model, original contents can be retrieved from imperfect information on memorized contents. This is a function that looks like the behavior of recollection of our memory: content-addressable memory [2], which is quite different from the computer of a so-called von Neumann type. However, the stage where contents are made to be memorized in the network and the stage where they are made to be retrieved are completely separated, it is difficult to say to simulate a higher-order functions of our human brain. Considering these situations, recently, the researches of the programmable neural networks [3] are advanced.

In this study, it devises a new spatiotemporal coupled Lorenz model that consists of three temporal coupling coefficients $c_{1,2,3}$ and three spatial coupling coefficients $d_{1,2,3}$, and finds that self-organized phase transition phenomena appear through the controlling of the values of $c_{1,2,3}$ and $d_{1,2,3}$, and manifests that proposed model possesses certain emergent abilities through the models are mounted in the mutually connected neural network systems. It reports because it is able to be confirmed that the system can achieve the same kind of these functions and it is an autonomous system.

1.2 Spatiotemporal Coupled Lorenz Model

Two continuous-time autonomous dynamical systems $\boldsymbol{X}_a$ and $\boldsymbol{X}_b$ are considered in n-dimensional Euclidean space $\boldsymbol{R}^n$, $\dot{\boldsymbol{X}}_a = \boldsymbol{F}(\boldsymbol{X}_a)$, $\dot{\boldsymbol{X}}_b = \boldsymbol{F}(\boldsymbol{X}_b)$. Here, $\boldsymbol{F}$ is considered to be the Lorenz system for both with $n = 3$, $\boldsymbol{F} \stackrel{\text{def}}{=} (f_1\ f_2\ f_3)^T$, where individual vector components are $\boldsymbol{X}_a = (x_1\ x_2\ x_3)^T$, $\boldsymbol{X}_b = (x_4\ x_5\ x_6)^T$. Then, it has considered this to be a model with three nonlinear oscillators: $\{\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}\}$=$\{x_1 - x_4,\ x_2 - x_5,\ x_3 - x_6\}$ are coupled to each other by coupling of the coupled Lorenz model spatially as well that is a new model called a spatiotemporal coupled Lorenz (STCL) model. Where $0 < c_{1,2,3} < 1$ are temporal coupling coefficients, $0 < d_{1,2,3} < 1$ are spatial coupling coefficients.

$$\begin{pmatrix} \dot{x}_{1,4} \\ \dot{x}_{2,5} \\ \dot{x}_{3,6} \end{pmatrix} = \begin{pmatrix} \sigma\,(x_{2,5} - x_{1,4}) \\ x_{1,4}\,(r - x_{3,6}) - x_{2,5} \\ x_{1,4}\,x_{2,5} - b\,x_{3,6} \end{pmatrix} \pm \boldsymbol{D}^* \begin{pmatrix} x_4 - x_1 \\ x_5 - x_2 \\ x_6 - x_3 \end{pmatrix} \tag{1.1}$$

$$\boldsymbol{D}^* = \boldsymbol{D} = \begin{pmatrix} c_1 & d_2 & d_3 \\ d_1 & c_2 & d_3 \\ d_1 & d_2 & c_3 \end{pmatrix} \text{:excitatory-excitatory connection} \tag{1.2}$$

$$\boldsymbol{D}^* = \tilde{\boldsymbol{D}} = \begin{pmatrix} c_1 & d_2 & 1 - d_3 \\ 1 - d_1 & c_2 & d_3 \\ d_1 & 1 - d_2 & c_3 \end{pmatrix} \text{:excitatory-inhibitory connection} \tag{1.3}$$

Figure 1.1 shows the behavior of $\{x_1 - x_4,\ x_2 - x_5,\ x_3 - x_6\}$ to change of the values of d at the value of certain c. Only $x_1 - x_4$ is illustrated in the figure which is plotted in $t = 0 \sim 10^5$ [sec], $\Delta t = 0.01$ [sec], $d = 0 \sim 1$. d is changing linearly with t where $d = 0.00001t$. Where, uniform spatial coupling coefficient $d_1 = d_2 = d_3 = d$ and uniform temporal coupling coefficient $c_1 = c_2 = c_3 = c$ are considered. As shown in this figure, when the excitatory-inhibitory connection matrix is used (EIC model), the domain of d separates to two places where the $\{x_1 - x_4,\ x_2 - x_5,\ x_3 - x_6\}$ does not synchronize. Furthermore, in the domain of certain c, when only the value of d is changed, self-organized phase transition phenomena appear like: chaos → limit cycle → intermittent chaos → fixed point.

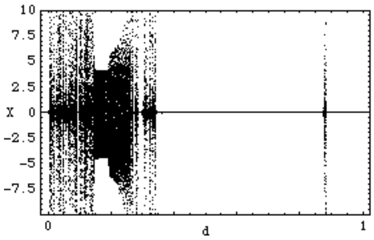

Figure 1.1: $x_1 - x_4$ versus d, excitatory-inhibitory connection, $c = 0.4$.

1.3 Subsystem by STCL with ACD

In the proposed model, it regards the three nonlinear oscillators: $\{X, Y, Z\}$=$\{x_1-x_4, x_2-x_5, x_3-x_6\}$ as three neurons. First, the synchronization phenomena are measured by the difference $\Delta_i(t) = |x_{i+3}(t) - x_i(t)|$, where $i = 1, 2, 3$. Then, the neurons are introduced using $\Delta_i(t)$ as

$$u_i(t) = \frac{1}{1 + \exp[-z_i(t)/z_0]}, \quad z_i(t) = \left(\frac{\varepsilon}{\Delta_i(t)}\right) - 1 \quad \text{:analog model,} \tag{1.4}$$

where the state of $u_i(t)$ have the continuous value [0, 1] of the ith neuron at time t, z_0 is the analog parameter, and ε is the criterion parameter of the synchronization [4][5]. This analog neuron becomes the binary neuron $u_i(t)$ which have two states of $\{0, 1\}$ if $z_0 \to 0$ then,

$$u_i(t) = \begin{cases} 1 & \text{if } \Delta_i(t) \le \varepsilon \\ 0 & \text{if } \Delta_i(t) > \varepsilon \end{cases} \quad \text{:digital model.} \tag{1.5}$$

Next, it introduces an abstract coincidence detector model (ACD model) which has been proposed by Fujii et al.[6]. The essences of this model: (1) Each neuron is an excitatory neuron which does not have memory but fires by the simultaneity of a momentary incidence spike. (2) It does not have an inhibitory neuron. (3) Network structure does not assume any specific structure. (4) All synaptic weight is set to one. (5) A certain transfer delay time which exists beforehand is between neurons. In this paper, it interprets this model to the Eq. 1.6 when the above (5) is neglected, where $w_{i0} = 1$ and proposed model's $k = 3$.

$$D_i(t) = \begin{cases} 1 & \text{if } N = \sum_{i=1}^{k} w_{i0}\, u_i(t) = k \quad \text{or } D = \prod_i w_{i0}\, u_i(t) = 1 \\ 0 & \text{if } N = \sum_{i=1}^{k} w_{i0}\, u_i(t) < k \quad \text{or } D = \prod_i w_{i0}\, u_i(t) \neq 1 \end{cases} \tag{1.6}$$

The spike trains are generated by using Eq. 1.5 from three neurons $\{X, Y, Z\}$, and the ACD output that is the output for them to have passed Eq. 1.6 is shown in Figure 1.2 as the ratio of the number of all generated spikes to all calculated steps, and as the ratio of the number of synchronized spikes of three neurons to all generated spikes. When the Figure 1.2 is compared with Figure 1.1, three

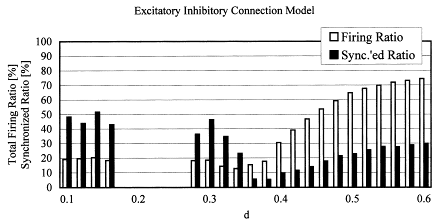

Figure 1.2: Histogram of the ratio of the number of all generated spikes to all calculated steps (white-bar) and the ratio of the number of synchronized spikes of three neurons to all generated spikes (black-bar) versus d of the EIC model, $c = 0.4$.

neurons $\{X, Y, Z\}$ have synchronized remarkably in the boundary regions of the limit cycle phase (which is a blank area in Figure 1.2) and each chaos phases. In other words, a high information processing ability is potential in these regions. Therefore, in the proposed system, the spatial coupling coefficients d_i of the STCL model is regulated dynamically by the following way like the Hopfield model,

$$d_i(t) = \sum_{j=1}^{n} w_{ij}\, u_j(t) - \theta_i(t), \tag{1.7}$$

where $\theta_i(t)$ is the threshold value, $w_{ij} (= w_{ji})$ is the synapic weight between ith and jth neurons and $w_{ii} = 0$, when $c_i(t) = constant$. Then, the difference of synchronization behavior by difference of the synaptic weight w_{ij} disappears almost according to this method. It is because the feedback of the spatial coupling coefficients $d_i(t)$ hang dynamically in chaotic attractor's behavior that is because this coefficients are controlled by using the spikes of the neurons $\{X, Y, Z\}$.

1.4 Emergent System Model

Proposed emergent system model consists of the n subsystems (n STCL models plus n ACD models) are connected to the external input of the mutually connected neural network through the connecting weight K_i with an unknown external vector. The state at the discrete time t of the ith neuron $v_i(t)$ is defined by

$$v_i(t+1) = sign\left[\sum_{j=1}^{n} J_{ij}\, v_j(t) + K_i\, k_i^{ext}\, S_i(t - \tau_{ij})\right], \tag{1.8}$$

$$S_i(t) = 2D_i(t) - 1. \tag{1.9}$$

The synaptic weight J_{ij} is given as the conventional autocorrelation associative memory,

$$J_{ij} = \begin{cases} \frac{1}{n} \sum_{\mu=1}^{p} \xi_i^{\mu} \xi_j^{\mu} (1 - \delta[i,j]) \\ J_{ji} \end{cases}, \tag{1.10}$$

to be embedded with p vectors ξ^{μ}, where ξ_i^{μ} are the ith components of ξ^{μ} which take binary value {-1, 1}, and k_i^{ext} is an external input vector which also takes binary value {-1, 1} and $0 < K_i < 1$ is external connecting weight. The ξ_i^{μ} are vectors memorized in the network beforehand, however, k_i^{ext} is an unknown vector for the network. Here, $sign[x] = 1$ $(x \geq 0)$ or -1 $(x < 0)$, $\delta[i,j] = 1$ $(i = j)$ or 0 $(i \neq j)$. And τ_{ij} are uniform random time delay to the synapse of each neuron.

There are two types in the proposed models. One of these is a digital-digital network model (a DDN model) that is mounted to the digital subsystems which consist of digital neuron model of Eq. 1.5, another system is an analog-digital network model (an ADN model) that is mounted to the analog subsystems which consist of analog neuron model of Eq. 1.4. Both these types are autocorrelation type associative memory models. Even though having no learning synapse weight systems in these models, the system shows several autonomous dynamics of retrieving embedded vectors by exciting an external input vector from the subsystems. Next section shows these results.

1.5 Dynamics of Proposed Model

1.5.1 DDN model

It shows the specifications for the numerical simulations. Concerning n subsystems which take the EIC models, $\sigma = 10$, $b = 8/3$, $r = 28$, $c_1 = c_2 = c_3 = 0.4$, $d_{1,2,3} = variables$, $w_{ij} = -1/3$, $\theta_i = -2/3$ and discrete time Δt for numerical simulations by Runge-Kutta method, $\Delta t = 0.01$ [sec]. Concerning a main system, number of neurons $n = 25$, number of embedded vectors $p = 3$. These embedded vectors ξ_i^{μ} and an external input vector k_i^{ext} are not orthogonal each other. And, this neural network system model's feature is in the desynchronized neuron model such as Hopfield model. In this paper, external connecting weight K_i takes constant value which is not depended by neuron index i, and time delay $\tau_{ij} = 0$ in DDN model.

The results are indicated in Figure 1.3. First, left figure which takes $K_i = 0.2$, the system behaves like an ordinary associative memory model. One of the embedded vector is retrieved by initial condition, and it is stabilized according to them. In this figure, this is a case of the system which retrieves the vector of $\mu = 1$. Next, right figure which takes $K_i = 0.9$, an external input vector k_i^{ext} and the reversed vector are repeated irregularly. Any embedded vectors are not retrieved. Then, middle figure which takes $K_i = 0.7$, all embedded vectors are irregularly retrieved by the external stimulation. Specifically, the embedded vectors $\mu = 1, 2, 3$ are retrieved at $t = 2, 10, 32$ [sec].

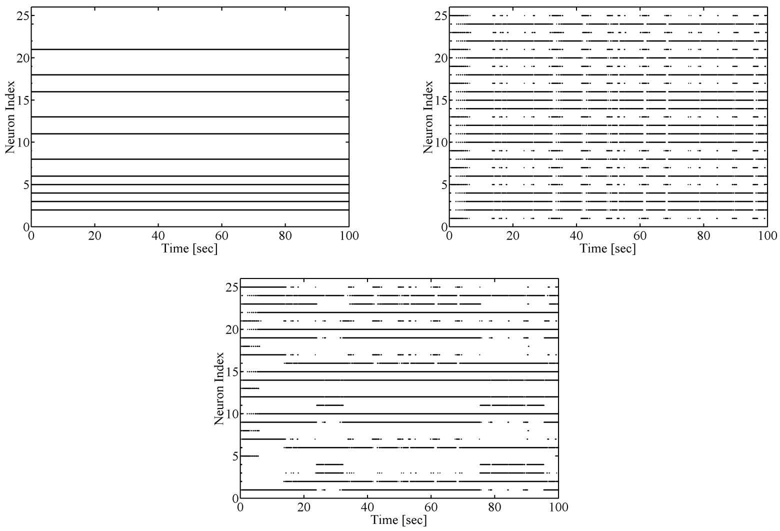

Figure 1.3: Retrieving dynamics of 25 neurons DDN model, left to right: $K_i = 0.2, 0.7, 0.9$, $t = 0 \sim 100$ [sec].

1.5.2 ADN model

Concerning ADN model, n subsystems which also take the EIC models and almost same parameters with DDN model. Only the criterion parameter takes $\varepsilon = 0.02$ so that frequency of the spikes from subsystem might almost become equal to DDN model and the analog parameter takes $z_0 = 0.01$. And it introduces the uniform random time delay τ_{ij} to the synapse of each neuron. These values of time delay are between from 10 to 250 [ms].

The results are indicated in Figure 1.4. The recollection of each memory is more clearer than DDN model. What should make a special mention here, the synaptic connection weight J_{ij} on the network has not been renewed in this model at all. Generally, in the associative memory systems, if certain learning algorithms are not added to Eq. 1.10 by using the plasticity of the synapse to retrieve two or more embedded vectors in the time series, such as autonomous dynamics of retrieving are not caused. Proposed system is a model of the autocorrelation associative memory. However, these embedded attractors can be made unstable by external stimuli. In other words, the trajectories cannot stay for a long time on the one attractor and wander between these embedded attractors by external stimuli which are generated by subsystems that consist of STCL models plus ACD models. This is a phenomenon that looks like "chaotic itinerancy" that Tsuda [7] discovered. Last, it shows the dynamics of four layers multi-moduled neural network which consists of the proposed ADN model in Figure 1.5.

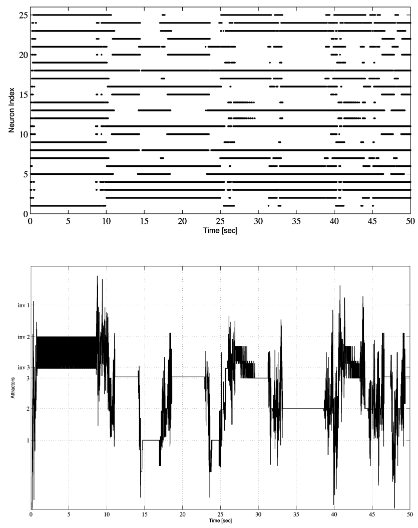

Figure 1.4: Upper: Retrieving dynamics of 25 neurons ADN model with uniform random time delay ($\tau_{ij} = 10 \sim 250$ [ms]) to the synapse of each neuron. Lower: Chaotic itinerancy between attractors $\mu = 1, 2, 3$. $K_i = 0.72$, $t = 0 \sim 50$ [sec].

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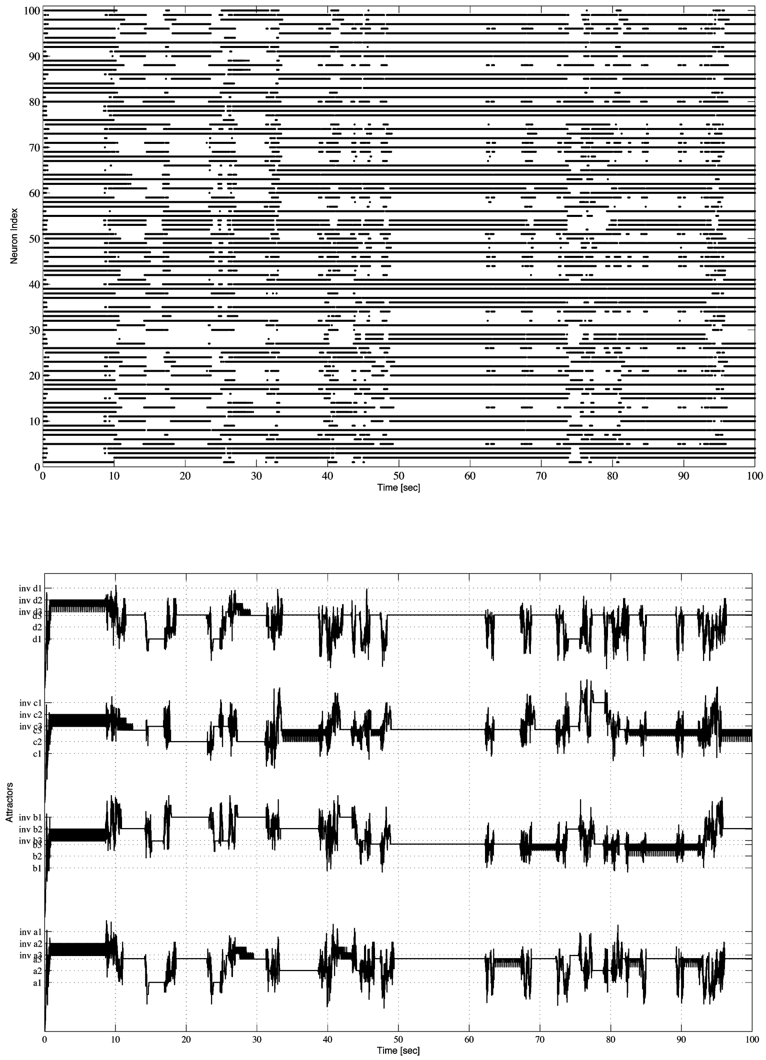

Figure 1.5: Upper: Retrieving dynamics of four layers 100 neurons ADN model with uniform random time delay ($\tau_{ij} = 10 \sim 250$ [ms]) to the synapse of each neuron. Lower: Chaotic itinerancy between attractors $\mu = 1, 2, 3$ in each four layer a, b ,c, d. $K_i = 0.72$, $t = 0 \sim 100$ [sec].