

Logic and Generalization

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Is formal logic a valid description of the world? I argue that it can't be. Let's start from an example.

The statement: "Green traffic lights mean go," is a simple statement that seems true. There are many ways it might not be true, such as when traffic lights are broken, or a car is stopped in the intersection.

There is a much more basic problem with the idea of such a truth that I want to focus on. When we see a green traffic light, we also see many other things about the existing conditions. We can imagine the entire image at a particular moment, with the visual effects of buildings, cars, pedestrians. The conclusion that green means go not only requires us to identify the green light as the determining factor in whether to go or not, but also requires us to eliminate from consideration all of the other pieces of information in the visual field.

Every time we drive up to an intersection the experience is different from every prior experience. How can we decide to drive through the intersection? Consider describing this in a logical framework. Many visual fields that contain a green light and the imperative to drive must be assigned a logical value "True." People don't know that this specific statement is true because they have been told it is true. Indeed, the statement that it is true has never been made before by anyone. How can it work?

In order to do this, our brains map many possible visual fields onto one state. That state represents the essential information about the visual field that we need to focus on (see figure 1). This convergence of the possible states of the world onto a much smaller set underlies how people generalize to deal with new conditions that we encounter in the world.

When convergence happens, the driver identifies truth of driving forward in this specific context by its sharing of attributes with previously encountered visual fields. Visual fields that converge in the brain onto a state that is associated with "truth" are considered true. This is a method of generalization from example that is quite different from the way logical inference works.

Still, it is a process of inference: If it is true about A it is true about A' because A' is similar to A. Similarity is not a statement of inference that is included in traditional logic and therefore logic doesn't encompass human inference. We immediately see why it would not be included in logic because similarity does not guarantee it would be true. We might make a mistake. For example, when a pedestrian is crossing the street, and the light is green, somebody might drive forward with catastrophic results. The visual field is similar in many ways to what it would be without the pedestrian. A mistake was made.

While similarity isn't part of formal logic, it is how people think. We can see this from the many ways people violate the qualities of logic. We call them the "Association fallacy," "Stereotyping," "Hasty Generalization,"

"Transference," "Guilt by association," and "Honor by association." Each of these qualities results in people making errors in identification of truths.

The way the "Association fallacy" is described logically is that something is a member of a class of things and has a property. Someone then thinks all members of the class have that property. By convergence, two things that share something in common are thought to share other properties. This might be true, but it is not necessarily true. It is not, therefore, true in traditional logic.

It is also apparent that these ways of thinking are not consistent between people—they differ from individual to individual. We say that they are not "universal." Different people will make different inferences about similarity and the properties that things share. This means that one of the most basic points of logic, to ensure that two rational people will arrive at the same truths, is not satisfied.

Why then do human beings use this kind of inference? Because it is necessary. The future is not exactly like the past. It isn't possible to know all truths and falsehoods so generalization from experience to similar conditions is necessary. Driving into an intersection is one example of many contexts where generalization is needed. The essence of cognition is to act in new conditions based upon prior experience.

Convergence of similar conditions is what allows for generalization.

We can say that real world truths are generalizations obtained from limited information, therefore they are theories about conditions that haven't been observed. What is determined to be similar (proximity) is different among different people. To prove truth by generalization, one must engage in the challenge of identifying which proximity map eliminates "irrelevant details" and keeps the relevant ones—a difficult and complex task.

A formal proof of these statements can be made by an example of the inability to describe a particular truth using a finite amount of information and to communicate it in finite time. Consider truth as a binary function on a set of points that are isomorphic to the sphere, \mathbb{S}^3 , with a closed stochastic fractal boundary between true and false values. This may be considered a model of the real world truth: This is where the ocean is on earth.

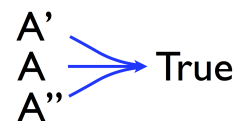


FIG. 1: True and false by generalization: Convergence in dynamics of experience identifies truth by association.